

”The globular cluster distributions in the Galaxy, M31 and M87: are many globulars disappeared to the galactic centres?”

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Abstract. The radial distribution of globular clusters in our Galaxy, M31 and M87 is studied and compared with that of halo stars. The globular cluster distributions seem significantly flatter than those of the stars bulge. Assuming this is a consequence of an evolution of the globular cluster distribution in these galaxies, a comparison with the (unevolved) stellar distribution allows us to obtain estimates of the number and total mass of clusters lost, which are possibly gone to feed the massive central objects present in these galaxies.

It results that the cluster systems studied should have been initially about one third and one forth richer than now in our Galaxy and in M31, respectively, and twice as abundant in M87. The estimated mass of globular clusters lost is compatible with the galactic nucleus masses.

Key words: galaxies: star clusters— galaxies: haloes— galaxies: evolution— galaxies: nuclei

1. Introduction

The globular cluster systems (GCS) in the two Virgo giant galaxies M87 and M49 are clearly less concentrated than the halo star distribution (Harris, 1986). Probably this feature is not common to all galaxies, even though in many cases the available data are probably not good enough to compare reliably the cluster and halo distributions. Anyway, a safe statement is that no case has been found where the GCS is more centrally concentrated than the halo (Harris, 1991) This is confirmed by the recent HST WFPC2 observation of 14 elliptical galaxies (Forbes et al 1996).

A possible explanation of this difference in the distributions has been suggested by Harris & Racine (1979) and by Racine (1991) as a difference in the formation ages of halo stars and globular clusters. Following these authors,

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globular clusters are formed earlier, when the density distribution was less peaked. This possibility cannot be ruled out, however it is not supported by any evidence of a significant older age for globular clusters respect to the halo: this age difference should be large enough to have allowed the mother galaxy to contract enough to form halo stars in a distribution as more concentrated than globulars as observed. Note that in disk galaxies the epoch of cluster formation could be early enough to force chemical enrichment but not to take on a distinct spatial structure (Harris 1986). Moreover, this picture does not explain why the tails of the two density distributions are about the same. Probably, a simpler explanation, working in the majority of cases, is the coeval birth of globular clusters and halo stars with a further evolution of the GCS radial distribution, while the collisionless halo stands almost unchanged. The causes of evolution are dynamical friction and tidal interaction with a compact nucleus; these phenomena can act to deplete the GCS in the denser inner galactic regions so to modify the initial radial distribution just in the central region leaving unchanged the outer profile, which remains similar to that of the halo component. If this is true, the halo radial profile clearly represents the shape of the initial cluster distribution.

2. Globular cluster system evolution

There are various indications that the GCS in a galaxy does not behave like a dissipationless system, as the halo component is. Actually even if the galaxy is not a spiral where disk shocking is an important cause of evolution, the tidal shock due to the passage near to the galaxy centre, and dynamical friction (which acts so to bring massive clusters closer and closer to the centre) are relevant causes of GCS evolution. This leads to a more or less important change of the GCS spatial distribution and mass function. The relevance of the mentioned phenomena depends on the galaxy characteristics: triaxiality enhances the efficiencies of both of them (Ostriker, Binney & Saha, 1989; Pesce, Capuzzo-Dolcetta & Vietri, 1992; Capuzzo-

Dolcetta 1993 (hereafter CD); Capuzzo-Dolcetta 1996). In particular, Pesce *et al.* (1992) showed that clusters on box orbits in a triaxial potential lose their orbital energy at a rate one order of magnitude larger than on loop orbits of comparable size and energy (even if they are quite elongated). It is very likely that a large fraction, if not all, of globular clusters are actually moving on box orbits and certainly not on quasi-circular loops, due to their early formation during the almost radial proto-galaxy collapse (see Binney 1988). So previous evaluations of dynamical friction efficiency based on clusters moving on circular orbits were undoubtedly *over-simplified* and leading to significantly overestimated values of the dynamical braking time-scales. This means that massive globulars in triaxial galaxies have probably suffered a lot of dynamical braking and have reached the centre of the mother galaxy were they can merge to form a super-massive object (not necessarily a black hole) or can feed a pre-existent one. Of course this nucleus, if massive enough, can act in a way to shatter incoming globulars before they are totally orbitally decayed CD examined in details the two contemporary effects and found that, assuming as typical globular cluster masses 10^5 , 10^6 , and $10^7 M_{\odot}$, nuclei as massive as $5 \times 10^6 M_{\odot}$, $2 \times 10^8 M_{\odot}$ and $5 \times 10^9 M_{\odot}$ are, respectively, needed to effectively halt the infall of globular clusters to the potential minimum.

Two equally interesting scenarios (quantitatively supported in CD) are open: i) a triaxial galaxy without a primordial massive nucleus can drive the merging of a significant mass in the form of orbitally decayed massive globulars in a time scale of the order of few 10^8 yrs, eventually leading, with modes which are not trivial to be studied, to a central object massive enough to stop further mass infall; ii) a moderately massive primordial nucleus is fed by decayed globulars such to produce a gravitational luminosity in the range of normal AGNs and to grow furtherly until a steady state is reached. If the mass of the primordial nucleus is large enough, dynamical friction on globular clusters is overwhelmed by the tidal shattering, and, moreover, the massive central object also changes the orbital structure around it.

2.1. Is dynamical friction an effective cause of GCS evolution?

The actual role of dynamical friction in galaxies has been questioned on the basis of various arguments which actually apply just to CGSs in M31 and M87, two of the best studied cases. It is worth nothing that even if it is plausible that its role is important whenever clusters move on almost radial orbits, the quantitative definition of the relevance of dynamical friction and tidal destruction has been done just in a triaxial galaxy (Pesce *et al.* 1992; CD; Ostriker *et al.* 1989).

Now, while it is now quite accepted that the inner part of M31 is triaxial (Lindblad 1956; Stark 1977; Bertola *et*

al. 1991) and so it is a galaxy where dynamical friction and tidal disruption effects should be significantly enhanced, a triaxiality for M87 is not evident. Good CCD photometric data by Zeilinger *et al.* (1993) for the inner M87 region show almost round isophotes in the inner M87 region ($r < 3''$) and a twisting occurs at $3''$ from the centre, the major axis being shifted to a position perpendicular to the jet and the ellipticity grows up to 0.2 at $r \simeq 80''$. This means that M87 is not necessarily one of the best candidate to investigate about the evolution of CGS distribution.

With regard to M31 two serious observational points are: i) the M31 galactic nucleus seems to be significantly redder than globular clusters (Surdin and Charikov, 1977), ii) M31 globular clusters seem to show a trend of increasing metallicity toward the galactic centre (Huchra *et al.* 1991), anyway this trend, see Fig. 2 in van den Bergh (1991) is quantitatively questionable.

Let us explain why in our opinion points i) and ii) are much less serious indication against the importance of dynamical friction and GCS evolution mechanism than it is superficially thought.

First of all the above mentioned data do not constitute a significant sample to extract general conclusions, referring just to one galaxy, anyway a trend of redder integrated colours towards the centres is a common feature of many galaxies (Gallagher *et al.* 1980), and needs in any case an explanation.

According to various authors, due to the apparent high metallicity of central region of M31, the decayed high mass clusters should have been more metal abundant (redder) than the only presently observed, and this needs a correlation between mass and metallicity for globular clusters. This metal abundance-mass correlation for globular clusters is claimed to be *ad hoc* because of the poor correlation presently observed between metal content and total *luminosity* of galactic globulars. There is an important caveat before concluding that also a Z mass-correlation is not holding. It comes by the fact that the Z -L correlation cannot be considered exactly representative of a Z mass-correlation because the mass-luminosity ratio actually depends on the metal content, and it increases with Z .

This means that a flat Z -L correlation transforms into a (more or less steep) *increasing* $M(Z)$. Moreover, what is observed now is the *present* luminosity-Z correlation, which, in the case of efficiency of the dynamical friction braking is biased (respect to the initial) towards lower luminosities (masses) and metallicities, likely hiding an initial stronger correlation.

A mass-metallicity relation for globular clusters is, anyway, not merely an *ad hoc* hypothesis, being verified for instance, in galaxies: the brighter galaxies contain redder, in the average, globulars (see van der Bergh 1991 for M31 and our Galaxy's clusters) and has a physical interpretation on the basis of a steeper, with increasing mass,

potential well to be overcome by the enriched material expelled by SNs.

Another point that seems hardly compatible with the claimed efficiency of dynamical friction (acting more on massive globulars) is that in M87 no dependence of the globular cluster luminosity function on galactocentric distance is found.

2.2. The radial dependence of the GCS luminosity function

There are various reasons why the lack of evidence of a radial dependence of GCS luminosity function is not a significative point against dynamical friction to be occurred on clusters: i) even if a spatial trend of the luminosity function is present (massive globulars moved to inner regions) it is expected to occur in quite central regions (within the bulge star core radius, see Capuzzo-Dolcetta and Tesseri 1996) not easily covered by proper observation; ii) projection effects weaken any radial trend of the luminosity function; iii) dynamical friction reduces the average galactocentric distance of massive GCs more than that of light globulars, but this contemporarily means they are shifted to inner galactic zones where they likely lose their individuality because they become hardly observable and more easily destroyed by the intense tidal field.

Let us give some quantitative support to point ii). Suppose to have a sample of globular clusters whose mean mass $\langle m \rangle$ varies with the galactocentric distance in a way to have smaller masses in external regions ($\langle m \rangle$ varies from $10^6 M_\odot$ to $10^5 M_\odot$ going from the centre to 5 times the core radius) and with a mass spectrum corresponding to a gaussian V-magnitude function characterized by a dispersion around the mean magnitude ($(M/L)_V = 1.6$ is assumed) which is larger ($\sigma_V^2 = 3$) in peripheral galactic regions than around the centre ($\sigma_V^2 = 1.5$) (this is what qualitatively expected when dynamical friction and tidal disruption have been effective). Assuming the GCS distributed spherically according to the modified Hubble profile

$$n(r) = n_0 \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-\frac{3}{2}} \quad (1)$$

we can compare the volume LF with the projected LF, sampled at various distances from the centre in a galaxy with a distance modulus $(m - M)_V = 31.3$ (similar to Virgo cluster) (see Fig. 1 a,b).

Projection should reduce the difference among the peak magnitudes and the widths of the LF sampled at various galactocentric distances. Actually, Fig. 1 shows how a V-peak difference of 4 mag reduces to just 2.5 mag, while the width of the projected LF results almost constant (independent on the galactocentric distance).

We conclude that a radial dependence of the width of such LFs would be undetectable, while the detection of a variation in the V-peak would require globular cluster sampling

well within the galaxy core, because (as it is seen in Fig. 1 b) the V-peaks of the LF sampled at $r = r_c$ and $r = 5r_c$ differs for a quantity similar to the standard deviation of the mean ($\sigma_\mu \simeq 0.1$ mag for a typical total sample of ≈ 500 clusters). This explains why, even if evolutionary effects have been active on GCSs, LF radial trends have not been detected in the past. Higher resolution observations to have larger sample abundances in inner galactic regions are needed.

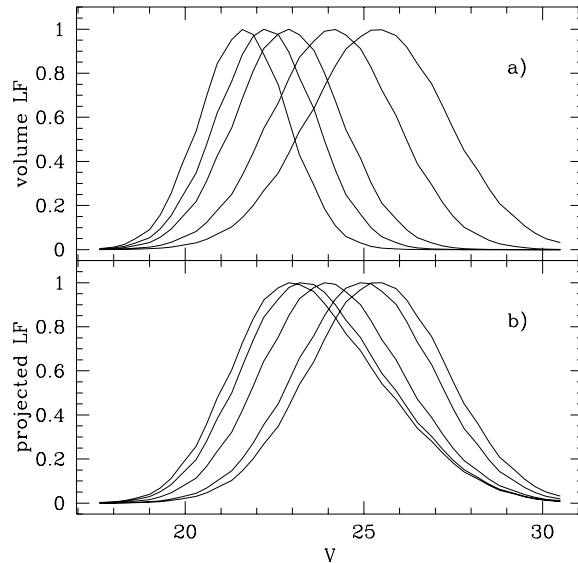


Fig. 1. panel a: normalized V-magnitude volume luminosity functions of the GCS at various galactocentric distances ($r/r_c = 0, 0.25, 0.5, 1, 5$); panel b: projected luminosity functions at the same (projected) galactocentric distances as in panel a.

3. Present and initial radial distributions

A way to estimate the number of globular clusters lost during the evolution of a globular cluster system was suggested by McLaughlin (1995) who applied it to the M87 galaxy. This estimate is based on the assumption that the stellar bulge and the globular cluster distributions were initially the same (due to a co-eval formation) and on that the shape of the stellar distribution has remained unchanged. The first hypothesis is supported by the observed similarity of the stellar bulge and globular cluster projected profiles in the outer regions (outside a certain distance \bar{r} from the centre) of various galaxies.

The second hypothesis stands firmly on that the bulge is a collisionless system. The initial globular cluster distribution $n_0(r)$ (assumed to be equal in shape to the present stellar distribution) is, practically, obtained by a scaling of the present GCS distribution to the stellar one. Of course

also the projected initial density profiles, $\sigma_0(r)$, are assumed to be the same. Once $n_0(r)$ or $\sigma_0(r)$ have been determined, the number of missing clusters is given by the integral over the whole galaxy of the difference between n_0 and n (or σ_0 and σ).

3.1. The data sets and interpolations

We consider the well established data sets of CGS in our Galaxy, M31 and M87, this latter mainly for the sake of a comparison with McLaughlin's (1995) results.

To fit the stellar distribution of M31 and M87 we used the model of de Vaucouleurs(1958) and for our Galaxy the Young's model (1976); good fits to the globular cluster distributions are obtained by means of the empirical King's models (King 1962).

To obtain the initial globular cluster distribution we vertically shift the stellar distribution to match, in the external region ($r \geq \bar{r}$), the GCS distribution, so to have a scaling factor, d , depending on \bar{r} . Thus, we can represent the initial globular cluster distribution by:

$$\log n_0 = \log n_s + d(\bar{r}) \quad (2)$$

$$\log \sigma_0 = \log \sigma_s + d(\bar{r}) \quad (3)$$

were $n_s(r)$ and $\sigma_s(r)$ are the (observed) stellar distribution volume and surface densities. Hereafter the volume and surface number densities will be given in kpc^{-3} and kpc^{-2} respectively.

4. Number of clusters missing in our Galaxy and M31

4.1. Our Galaxy

The most complete set of data of galactic globular clusters is still given by Webbink (1985). It refers to 154 globular clusters. For our purposes we need only the distance of each cluster from the galactic centre. A good fit to the present distribution is:

$$n(r) = \left\{ \left[\frac{1}{1 + (\frac{r}{2})^2} \right]^{0.5} - \left[\frac{1}{1 + (\frac{b}{2})^2} \right]^{0.5} \right\}^3 \quad (4)$$

where $b = 49$.

As distribution of bulge stars in our Galaxy we use the Young's model (1960) (see Fig. 2). Thus we obtain the initial globular cluster distribution:

$$n_0(r) = 426 \exp \left[-7.669 \cdot \left(\frac{r}{2.7} \right)^{0.25} \right] \left(\frac{r}{2.7} \right)^{-0.875} \quad (5)$$

The estimated number of globular clusters lost for our Galaxy is $N_l = 56$, i.e. about 36% of the present sample's abundance.

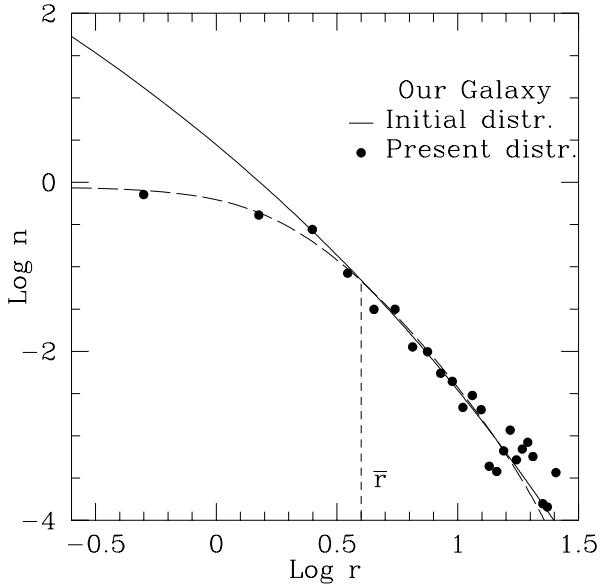


Fig. 2. The globular cluster initial distribution and the present one for our Galaxy. The dashed line refers to the point where the vertically shape bulge and GCS profile start to overlap

4.2. Andromeda (M31)

Various compilations of data for globular clusters in M31 are available. We refer to the "Adopted Best Sample" of Battistini *et al.* (1993) compilation, for it is the most complete and best discussed source of data for the radial distribution of globular clusters in this galaxy.

The flattening of the M31 globular cluster distribution compared to that of the star spheroidal component was first noted by de Vaucouleurs & Buta (1978), later questioned by Wirth, Smarr & Bruno (1985) who claimed a large part of this flattening as being due to incompleteness. The completeness of the Battistini *et al.* data in the inner bulge region is well addressed, and a residual flattening of their adopted samples respect to the spheroidal star component is evident.

Matching the globular clusters data with a King's model leads to the following analitycal fit to the present globular cluster distribution:

$$\sigma(r) = 16.7 \left\{ \left[\frac{1}{1 + (\frac{r}{0.7})^2} \right]^{0.5} - \left[\frac{1}{1 + (\frac{a}{0.7})^2} \right]^{0.5} \right\}^{1.52} \quad (6)$$

were $a = 30$. The data for the stellar distribution are taken from de Vaucouleurs (1958), taking the luminosity distribution as representative of the stellar density distribution (see Fig. 3). We obtain the initial globular clusters distribution:

$$\sigma_0(r) = 785.2 \cdot \exp [-7.427 \cdot r^{0.2} + 3.1] \quad (7)$$

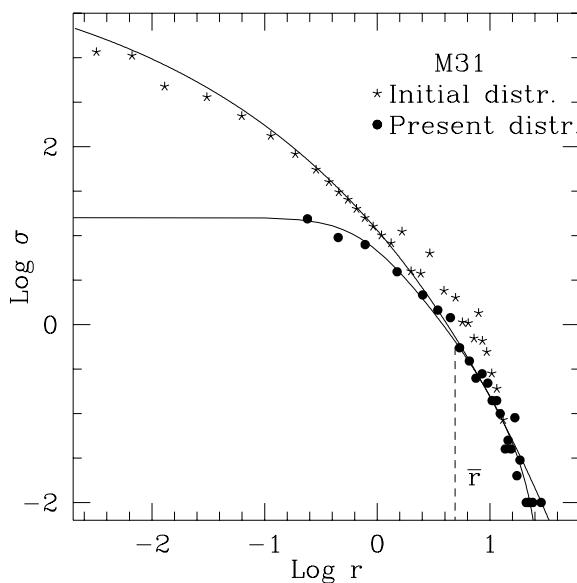


Fig. 3. Initial and present globular cluster distributions in M31

The surface integral of $\sigma_0(r) - \sigma(r)$ gives $N_l \simeq 76$ as number of globular clusters lost; this is 25% of the present number.

4.3. Evaluation of globular cluster mass fallen to the galactic centres

An approximate value of the mass fallen to the centre of M31 and our Galaxy can be given by mean of the knowledge of N_l and of the average mass of destroyed globular cluster $\langle m_l \rangle$. The determination of $\langle m_l \rangle$ requires a detailed evaluation of the tidal disruption and dynamical friction effects on an assumed initial mass function. By the way, due to that the two phenomena erode the CGS on apposite sides of the mass function, the mean value of the globular cluster mass $\langle m \rangle$ is not expected to change very much in time whenever the initial mass function is not too asymmetric, and thus it can be chosen as a good reference value for $\langle m_l \rangle$. This is confirmed by results obtained with a theoretical model (Capuzzo-Dolcetta & Tesserri 1996) under the hypothesis of a constant initial mass function ($\Phi(m) = \text{constant}$).

The knowledge of the mean mass of globular clusters, $\langle m \rangle = 3.2 \cdot 10^5 M_\odot$ for our Galaxy and $\langle m \rangle = 2.7 \cdot 10^5 M_\odot$ for M31, gives as mass lost $M_l = 1.8 \cdot 10^7 M_\odot$ and $M_l = 2.1 \cdot 10^7 M_\odot$ respectively. These values should be compared with the nucleus masses in our galaxy ($3 \cdot 10^6 M_\odot$, see Krabbe et al 1995) and in M31 ($10^7 M_\odot$, see e.g. Melia 1992).

4.4. Sources of error

A significant source of error evaluation of N_l is the indetermination in the of the region where the globular cluster

distribution profile is evolved, i.e. the estimate of \bar{r} . A relative error in \bar{r} induces an error in $N_l(\bar{r})$:

$$\frac{\Delta N_l}{N_l} = \frac{\partial N_l}{\partial \bar{r}} \frac{\bar{r}}{N_l} \frac{\Delta \bar{r}}{\bar{r}}, \quad (8)$$

being:

$$\frac{\partial N_l}{\partial \bar{r}} = \ln 10 \cdot 2\pi \cdot 10^{d(\bar{r})} \cdot d'(\bar{r}) \int_0^{\bar{r}} \sigma_s(r) r dr \quad (9)$$

or, when the spatial density is available:

$$\frac{\partial N_l}{\partial \bar{r}} = \ln 10 \cdot 4\pi \cdot 10^{d(\bar{r})} \cdot d'(\bar{r}) \int_0^{\bar{r}} n_s(r) r^2 dr \quad (10)$$

where $d'(\bar{r})$ is the derivative of $d(\bar{r})$ with respect to \bar{r} . For M31 and our Galaxy we find that the error $\Delta \bar{r}/\bar{r}$ reflects in relative errors $\Delta N_l/N_l$ given by $0.75\Delta \bar{r}/\bar{r}$ and $0.63\Delta \bar{r}/\bar{r}$, respectively.

5. M87

For the sake of comparison with previous work (McLaughlin 1995) we applied our method to M87. The data are taken from McLaughlin (1995) and from de Vaucouleurs and Nieto (1978, 1979) for globular clusters and halo stars, respectively. The fits we obtained from those distributions are:

$$\sigma(r) = 15.5 \left\{ \left[\frac{1}{1 + (\frac{r}{1.6})^2} \right]^{0.5} - \left[\frac{1}{1 + (\frac{c}{1.6})^2} \right]^{0.5} \right\} \quad (11)$$

$$\sigma_0(r) = 67.62 \cdot \exp \left[-3.848 \left(\frac{r}{1.543} \right)^{0.349} + 3.875 \right] \quad (12)$$

where $c = 60$.

We have fixed \bar{r} as the point where the two distributions clearly show the same shape, as it is shown in Fig. 4 ($\bar{r} = 12.5 \text{ kpc}$).

The number of globular clusters lost is found to be $N_l = 4018$, which is about the same number of globular clusters presently observed.

We have numerically evaluated the error induced on N_l by an error in \bar{r} , finding that an error $\Delta \bar{r}/\bar{r}$ reflects in a relative errors for $\Delta N_l/N_l \simeq 2.12\Delta \bar{r}/\bar{r}$; i.e. much greater than for our Galaxy and M31.

This large sensitivity of N_l on the choice of \bar{r} is part of the explanation of the great difference between our value of N_l and that given by McLaughlin ($N_l \simeq 1150$). This difference is indeed accounted by:

i) we have considered as best value for the radius $\bar{r} \simeq 12.5 \text{ kpc}$ instead of 8 kpc (used by McLaughlin 1995); it seems that the two distributions have a very similar shape only outside 12.5 kpc . The value of 8 kpc for \bar{r} seems an underestimate. Infact as shown by Fig. 5, the distribution obtained shifting vertically the bulge star profile

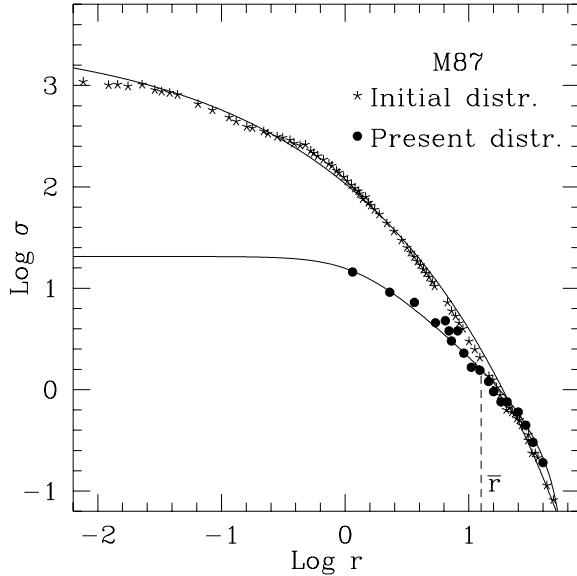


Fig. 4. The initial and the present globular cluster distribution in M87

to intersect the present GCS distribution at $\bar{r} = 8 \text{ kpc}$ represents an acceptable initial distribution for the GCS just if we accept as realistic that in external regions ($r \geq 8 \text{ kpc}$) there are at present more clusters than initially. This implies the existence of a mechanism which populated the external regions, while dynamical friction and tidal disruption depopulated the inner regions.

ii) the analytical fits are obtained with different functions: McLaughlin used, for both globular clusters and star bulge isotropic, single mass King's (1966) models, while we used an empirical King's (1963) model to fit the present globular cluster distribution and the de Vaucouleur's (1958) model (more peaked than the King's 1966 one) to fit the bulge distribution. Anyway the difference in the analytical fits can account for just a 30% difference in N_l . Infact, we obtain $N_l \simeq 1590$ instead of $N_l \simeq 1150$ integrating our distribution up to the same radius ($\bar{r} = 8 \text{ kpc}$) used by McLaughlin.

Now, if we take as mean mass of the globular clusters in M87 $\langle m \rangle = 6.6 \cdot 10^5 M_\odot$ (McLaughlin 1995) our estimate of the mass lost is $2.65 \cdot 10^9 M_\odot$. If we compare this value with the nucleus mass of M87 (which is estimated to be $\simeq 2.4 \cdot 10^9 M_\odot$ within 18 pc of the nucleus, see Ford and al 1994) we see, also in this case, that the disruption of globular clusters could have strongly influenced the formation of the central nucleus of this galaxy.

6. Conclusions

It is both a reasonable and simple hypothesis that the globular cluster and spheroidal components of galaxies formed contemporarily during the first stages of proto-

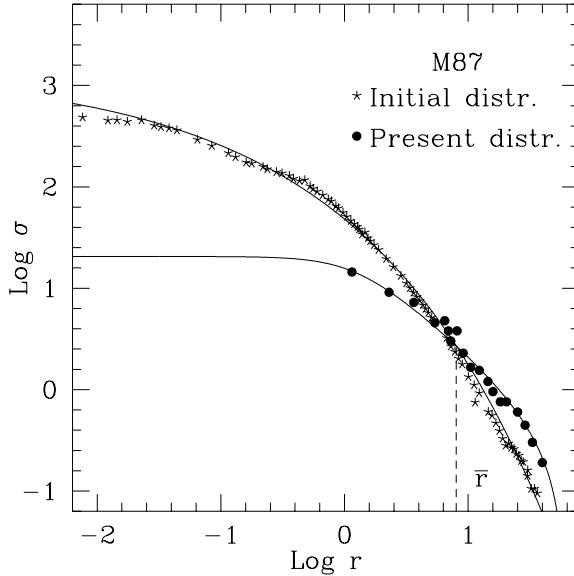


Fig. 5. As in Fig. 3, but with $\bar{r} = 8 \text{ kpc}$

galaxy collapse so to have, initially, the same spatial distribution. Observed (and kinematic) spatial differences should be explained on the basis of evolution of the globular cluster system (GCS).

We have given reference to quantitative studies which point out the role of dynamical causes of this evolution in galaxies where clusters move on sufficiently radial orbits. We have also explained why most of the observational data available is, at present, at all unsufficient to rule out that such an evolution occurred. To state something meaningful, observations of clusters in the innermost regions (i.e. within the bulge core) to compare with clusters in outer regions of their parent galaxy, as well as kinematic data to determine the cluster velocity ellipsoid are needed.

Through the comparison between the globular cluster and spheroidal radial distributions we determined the number, N_l , of clusters lost in our Galaxy, M31 and M87. We found that the GCSs of our Galaxy, M31 and M87 should have been initially 1.45, 1.25, and 2 times more populous than now.

The mass of missing clusters has likely gone to the centre of the parent galaxy, where it can contribute to enrich the nucleus by an amount of the order of $N_l \langle m \rangle$, where $\langle m \rangle$ is the (present) mean value of cluster mass. This corresponds to $1.8 \cdot 10^7 M_\odot$, $2.1 \cdot 10^7 M_\odot$ and $2.7 \cdot 10^9 M_\odot$ for the Galaxy, M31 and M87, respectively. It is relevant noting that these values are all very similar to available estimates of the nucleus masses in these galaxies.

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